tial problems, and the idea of perturbation methods [2] for the construction of the asymptotic expansions.

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STABILITY OF THERMOCAPILLARY MOTION IN A CYLINDRICAL LAYER

E. A. Ryabitskii

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The stability of thermocapillary motion in a planar layer and a liquid cylinder was studied in [1, 2]. In the present study we will consider the stability of thermocapillary convection in a cylindrical layer with an undeformed free surface. The effect of the ratio of cylinder radii on motion stability is considered. It is shown that for axisymmetric disturbances at certain values of the problem parameters, increase in relative thickness of the inner cylinder leads to reduction in stability.

1. We will consider a cylindrical layer of viscous thermally conductive liquid bounded by solid inner and free outer surfaces in the absence of gravity. We introduce a cylindrical coordinate system with the z axis directed along the cylinder directrix. The equation of the solid boundary is $r = r_0$. We assume that the free surface is cylindrical ($r = r_1$) and undeformed. The temperature dependence of the surface tension coefficient is given by $\sigma = \sigma_0 - \kappa(\theta - \theta_0)$.

Let the free surface be heated by a law $\theta_B = -Az$ (A is a specified constant value). Then the steady-state axisymmetric thermocapillary motion which develops due to change in surface tension will be described by the equations

$$\begin{gathered} u = v = 0,\ w = B_1(\xi^2 - d^2) + B_2 \ln(\xi/d),\ p_\eta = 4B_1, \\ \theta = -\eta - \mathrm{MaPr}\,[B_1(\xi^4 - 1)/4 - (d^2B_1 + B_2 + \ln d B_2)(\xi^2 - 1) + \\ + B_2(\xi^2 + d^2)\ln\xi + B_1 d^4 \ln\xi]/4, \end{gathered} \tag{1.1}$$

where the constants $B_1 = (1 - d^2 + 2\ln d)[(1 - d^2)(3 - d^2) + 4\ln d]^{-1}$, $B_2 = (1 - d^2)^2[(1 - d^2)(3 - d^2) + 4\ln d]^{-1}$ are found from the conditions of adhesion and closed flow

$$\int_d^1 \xi w(\xi)\, d\xi = 0. \tag{1.2}$$

Here and below, $\xi = r/r_1$; $\eta = z/r_1$; $d = r_0/r_1 < 1$; $\mathrm{Ma} = r_1^2\kappa A/\rho\nu^2$ is the Marangoni number; $\mathrm{Pr} = \nu/\chi$, the Prandtl number; $\mathrm{Bi} = \beta r_1/\lambda$, the Biot number; ν and χ, kinematic viscosity and thermal diffusivity coefficients; λ and β, thermal conductivity and interphase exchange coefficients; ρ, density. For units of length, time, velocity, temperature, and pressure we take r_1, $r_1^2/\nu\mathrm{Ma}$, $\nu\mathrm{Ma}/r_1$, Ar_1, and $\rho\nu^2\mathrm{Ma}^2/r_1^2$, respectively.

As $d \to 0$ the motion of Eq. (1.1) transforms to thermocapillary flow of a completely liquid cylinder: $u = v = 0$, $w = (\xi^2 - 0.5)/2$, $p_\eta = 2$, $\theta = -\eta - \mathrm{MaPr}(1 - \xi^2)^2/32$, the stability of which was studied in [2]. In [3] a stability study was performed for axisymmetric disturbances of a motion with logarithmic velocity profile which did not satisfy closure condition (1.2).

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We will now turn to study of the stability of the motion of Eq. (1.1). We seek a disturbance of the velocity, pressure, and temperature vector in the form

$$(U, V, W, P, T) = (U(\xi), V(\xi), W(\xi), P(\xi), T(\xi)) \times \exp[i\alpha\eta + im\varphi - iC\tau],$$

where α and m are axial and azimuthal wave numbers; $C = C_r + iC_i$ is the complex decrement, and τ is dimensionless time.

The linearized Navier–Stokes equations take on the form [4]

$$\begin{gathered} aU + \mathrm{Ma}\, P' = -i\alpha W' - \frac{im}{\xi^2}(\xi V)', \\ aV + \frac{im}{\xi}\mathrm{Ma}\, P = \left[\frac{1}{\xi}(\xi V)'\right]' + \frac{2im}{\xi^2}U, \\ aW + \mathrm{Ma}\, w_\xi U + i\alpha\, \mathrm{Ma}\, P = \frac{1}{\xi}(\xi W')', \quad (\xi U)' + imV + i\alpha W = 0, \\ bT + \mathrm{Ma}\,\mathrm{Pr}\,\theta_\xi U - \mathrm{Ma}\,\mathrm{Pr}\, W = \frac{1}{\xi}(\xi T')', \\ a = i\,\mathrm{Ma}(\alpha w - C) + \frac{m^2}{\xi^2} + \alpha^2, \quad b = i\,\mathrm{Ma}\,\mathrm{Pr}(\alpha w - C) + \frac{m^2}{\xi^2} + \alpha^2; \end{gathered} \tag{1.3}$$

with conditions on the rigid boundary:

$$\xi = d: \ U = V = W = T' = 0; \tag{1.4}$$

and the free surface:

$$\begin{gathered} \xi = 1: \ V' - V + imT = 0, \ U = 0, \\ W' + i\alpha T = 0, \ T' + \mathrm{Bi}\, T = 0. \end{gathered} \tag{1.5}$$

2. We will perform an asymptotic analysis of the problem of Eqs. (1.3)-(1.5) for long waves ($\alpha \to 0$). Let m = 0, whereupon the problem for the function V is separable. We expand the unknown quantities in a series in α: $C = C_0 + O(\alpha)$, $U = \alpha U_0 + O(\alpha^2)$, $W = W_0 + O(\alpha)$, $P = \alpha P_0 + O(\alpha^2)$, $T = \alpha^{-1}T_0 + O(1)$. The irregularity of the expansion of the temperature in α stems from consideration of thermal modes in Eq. (1.5). Substituting the given expansion in the original equations, solving and satisfying the boundary conditions, we obtain a characteristic equation for finding the decrement C:

$$\begin{gathered} \beta[J_0(\beta d)Y_1(\beta) - J_1(\beta)Y_0(\beta d)]\{\gamma[Y_1(\gamma d)J_1(\gamma) - \\ - J_1(\gamma d)Y_1(\gamma)] + \mathrm{Bi}[J_1(\gamma d)Y_0(\gamma) - Y_1(\gamma d)J_0(\gamma)]\} = 0. \end{gathered} \tag{2.1}$$

Here J_0, J_1, Y_0, and Y_1 are Bessel fucntions of the first and second kind, $\beta = \sqrt{i\mathrm{Ma}C_0}$, $\gamma = \sqrt{\mathrm{Pr}}\,\beta$. Equation (2.1) has an even number of real roots. For example, for d = 0.1, Bi = 2, $\beta_0 = 0$, $\beta_1 = 2.29$, $\beta_2 = 5.43$, $\gamma_1 = 2.41$, $\gamma_2 = 5.52$. Thus all the remaining eigenvalues are complex and negative. By taking subsequent terms of the expansion it can be shown that this statement is also valid for $\beta_0 = 0$. For $m \neq 0$ the characteristic equation has the form

$$\begin{gathered} [J'_m(\beta)Y_m(\beta d) - Y'_m(\beta)J_m(\beta d)]\{J'_m(\gamma d)Y'_m(\gamma) - Y'_m(\gamma d)J'_m(\gamma) + \\ + \mathrm{Bi}[Y_m(\gamma)J'_m(\gamma d) - J_m(\gamma)Y'_m(\gamma d)]\} = 0. \end{gathered} \tag{2.2}$$

The roots of Eq. (2.2) are also real. For example, for m = 1, d = 0.1, Bi = 0, $\beta_1 = 2.41$, $\beta_2 = 5.52$, $\gamma_1 = 2.32$, and $\gamma_2 = 5.06$. Consequently, the motion of Eq. (1.1) is stable relative to longwave perturbations.

3. The problem of Eqs. (1.3) and (1.4) was solved by the numerical orthogonalization method [5], while in light of satisfaction of Eq. (1.5) the method of secants was used to find the complex decrement. Calculation was begun from the asymptotic C values obtained as $\alpha \to 0$.

As a control, critical values of C_r and α were determined for small d. The values obtained for $d = 10^{-10}$, m = 0, Pr = 0.4, Ma = 1260.8, and Bi = 0, i.e., $C_r = 0.1204$, α = 2.49 agree well with the results of [2]: $C_r = 0.1156$, α = 2.45.

Critical values of the Marangoni number as a function of d were calculated for the same values of m, Pr, and Bi. Results are shown in Figs. 1 and 2. Here Ma_* is the minimum over

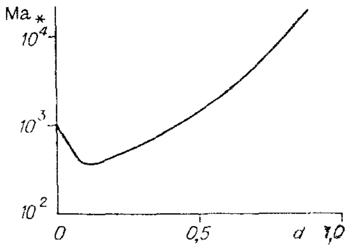

Fig. 1

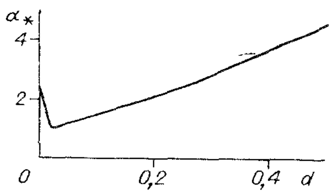

Fig. 2

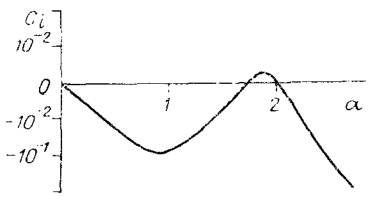

Fig. 3

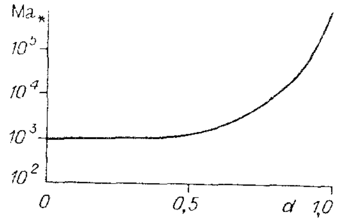

Fig. 4

α of the critical Marangoni number, while α_* is that value of the wave number at which this minimum is achieved. At d = 0 the critical Marangoni number is equal to 1260.8, which corresponds to the case of a completely liquid cylinder; with growth in d, Ma_* decreases, reaching its minimum value of 600 at d = 0.1. Further increase in d leads to monotonic increase in the Marangoni number, with the value of 1260 being reached again at d = 0.42. The critical wave number curve behaves in a similar fashion (Fig. 2), decreasing from $\alpha_* = 2.49$ at d = 0 to $\alpha_* = 1.03$ at $d = 2\cdot 10^{-2}$, and then increasing monotonically. Figure 3 shows the dependence of C_i on α for d = 0.1 and Ma = 600, values of $\alpha \in [1.74, 1.97]$ at which $C_i > 0$ correspond to the region of loss of stability of the original motion of Eq. (1.1).

Calculations performed for various values of Pr less than 0.4 showed qualitative agreement of the graphs of Ma_* vs. d with Fig. 1. Thus, for low Prandtl numbers the presence within the liquid of a thin cylindrical bar reduces the stability of the motion of Eq. (1.1) relative to axisymmetric disturbances.

The effect of the ratio of cylinder radii on motion stability was studied for fused germanium with Pr = 0.016. As in [2], it was found that in this case azimuthal (m = 1) disturbances are the most dangerous, stability being lost at negative α values. Figure 4 shows Ma_* vs. d at m = 1 and Bi = 0. It has been shown that the most unstable cylinder is a completely liquid one for which $Ma_* = 897$ and $\alpha_* = -0.147$, while with increase in d the critical Marangoni numbers increase monotonically.

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